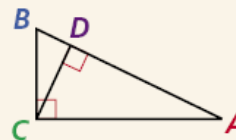


Theorem 8-1-1

The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

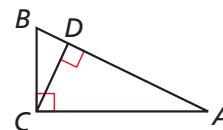
$$\triangle ABC \sim \triangle ACD \sim \triangle CBD$$

**PROOF****Theorem 8-1-1**

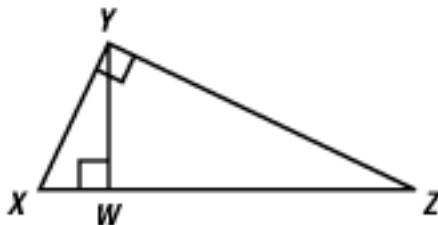
Given: $\triangle ABC$ is a right triangle with altitude \overline{CD} .

Prove: $\triangle ABC \sim \triangle ACD \sim \triangle CBD$

Proof: The right angles in $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$ are all congruent. By the Reflexive Property of Congruence, $\angle A \cong \angle A$. Therefore $\triangle ABC \sim \triangle ACD$ by the AA Similarity Theorem. Similarly, $\angle B \cong \angle B$, so $\triangle ABC \sim \triangle CBD$. By the Transitive Property of Similarity, $\triangle ABC \sim \triangle ACD \sim \triangle CBD$.

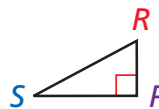
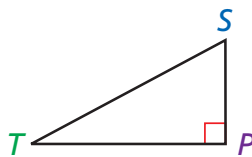
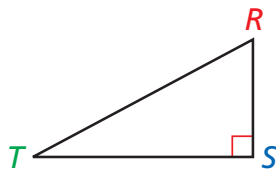
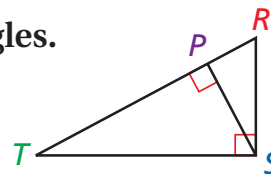
**Refer to video Example 1.**

Write a similarity statement comparing the three triangles.

**1****Identifying Similar Right Triangles**

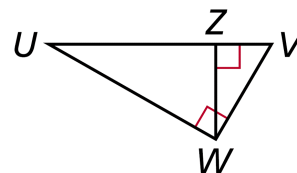
Write a similarity statement comparing the three triangles.

Sketch the three right triangles with the angles of the triangles in corresponding positions.

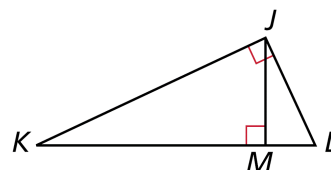


By Theorem 8-1-1, $\triangle RST \sim \triangle SPT \sim \triangle RPS$.

Example 1. Write a similarity statement comparing the three triangles.



6. Guided Practice. Write a similarity statement comparing the three triangles.



Consider the proportion $\frac{a}{x} = \frac{x}{b}$. In this case, the means of the proportion are the same number, and that number is the *geometric mean* of the extremes. The **geometric mean** of two positive numbers is the positive square root of their product. So the geometric mean of a and b is the *positive number* x such that $x = \sqrt{ab}$, or $x^2 = ab$.

Video Example 2. Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

A. 8 and 2

B. 5 and 15

2 Finding Geometric Means

Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

A 4 and 9

Let x be the geometric mean.

$$x^2 = (4)(9) = 36 \quad \text{Def. of geometric mean}$$

$$x = 6 \quad \text{Find the positive square root.}$$

B 6 and 15

Let x be the geometric mean.

$$x^2 = (6)(15) = 90 \quad \text{Def. of geometric mean}$$

$$x = \sqrt{90} = 3\sqrt{10} \quad \text{Find the positive square root.}$$

Example 2. Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

A. 4 and 25

B. 8 and 9

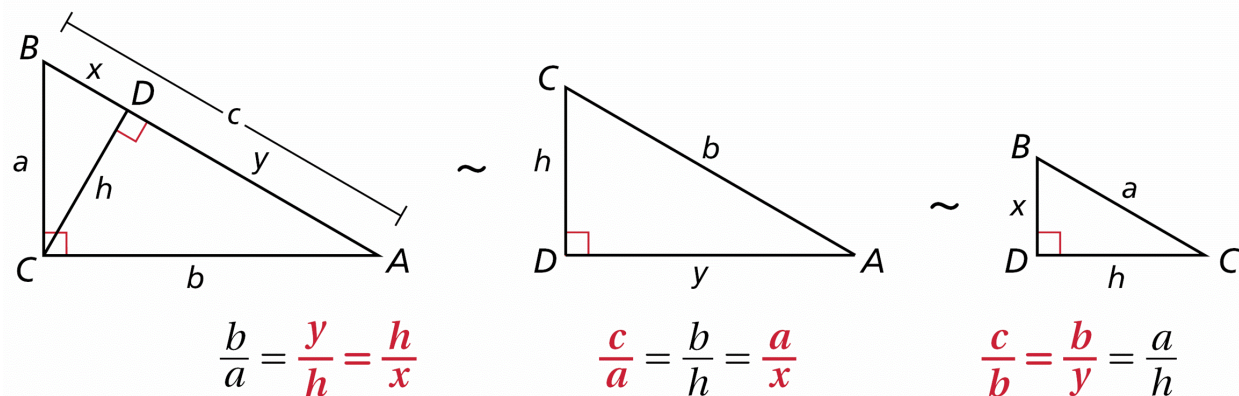
Guided Practice. Find the geometric mean of each pair of numbers. If necessary, give the answer in simplest radical form.

7. 2 and 8

8. 10 and 30

9. 8 and 9

You write proportions comparing the side lengths of the triangles formed by the altitude to the hypotenuse of a right triangle.

**Corollaries****Geometric Means**

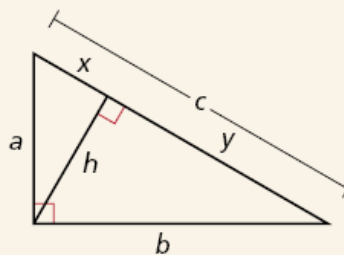
8-1-2 The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments of the hypotenuse.

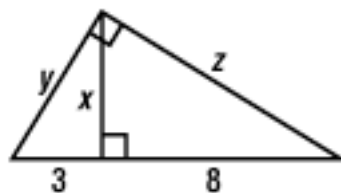
$$h^2 = xy$$

8-1-3 The length of a leg of a right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$a^2 = xc$$

$$b^2 = yc$$



Refer to video example Example 3.**Find x , y , and z .****Helpful Hint**

Once you've found the unknown side lengths, you can use the Pythagorean Theorem to check your answers.

3 Finding Side Lengths in Right TrianglesFind x , y , and z .

$$x^2 = (2)(10) = 20$$

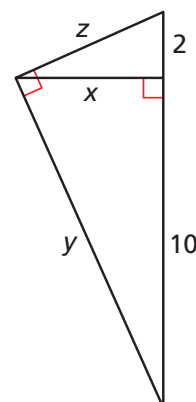
$$x = \sqrt{20} = 2\sqrt{5}$$

$$y^2 = (12)(10) = 120$$

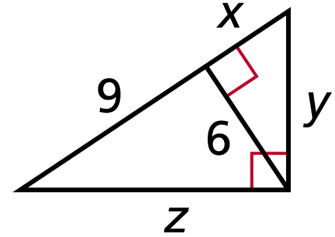
$$y = \sqrt{120} = 2\sqrt{30}$$

$$z^2 = (12)(2) = 24$$

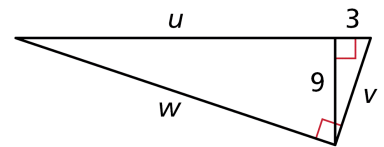
$$z = \sqrt{24} = 2\sqrt{6}$$

 *x is the geometric mean of 2 and 10.**Find the positive square root.* *y is the geometric mean of 12 and 10.**Find the positive square root.* *z is the geometric mean of 12 and 2.**Find the positive square root.*

Example 3. Find x , y , & z .

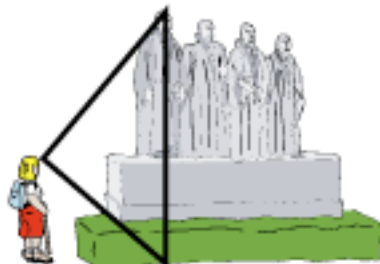


10. Guided Practice. Find u , v , and w .



Refer to video Example 4.

To estimate the height of a statue, Holly steps away from the statue until her line of sight to the top of the statue and her line of sight to the bottom of the statue form a 90° angle. Her eyes are 5 ft 6 in. above the ground, and she is standing 25 ft from the statue. How tall is the statue to the nearest foot?

**4 Measurement Application**

To estimate the height of Big Tex at the State Fair of Texas, Michael steps away from the statue until his line of sight to the top of the statue and his line of sight to the bottom of the statue form a 90° angle. His eyes are 5 ft above the ground, and he is standing 15 ft 3 in. from Big Tex. How tall is Big Tex to the nearest foot?

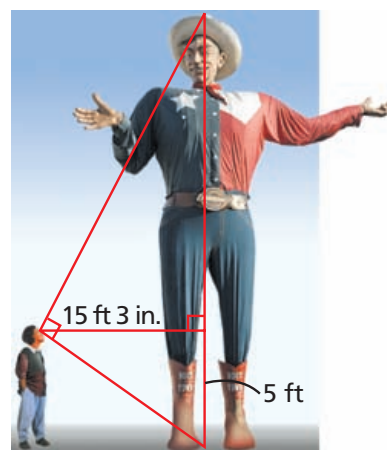
Let x be the height of Big Tex above eye level.

15 ft 3 in. = 15.25 ft *Convert 3 in. to 0.25 ft.*

$$(15.25)^2 = 5x \quad \text{15.25 is the geometric mean of 5 and } x.$$

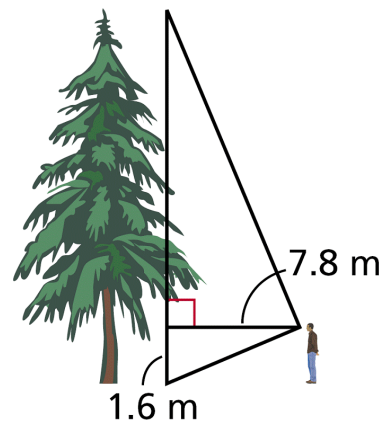
$$x = 46.5125 \approx 47 \quad \text{Solve for } x \text{ and round.}$$

Big Tex is about $47 + 5$, or 52 ft tall.

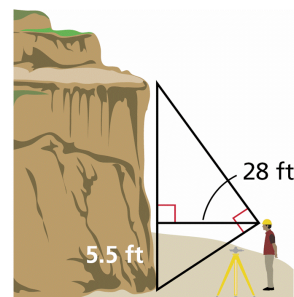


Not drawn to scale

Example 4. To estimate the height of a Douglas fir, Jan positions herself so that her lines of sight to the top and bottom of the tree form a 90° angle. Her eyes are about 1.6 m above the ground, and she is standing 7.8 m from the tree. What is the height of the tree to the nearest meter?



17. Guided Practice. A surveyor positions himself so that his line of sight to the top of a cliff and his line of sight to the bottom form a right angle as shown. What is the height of the cliff to the nearest foot?



8-1 Assignment (pp 537-539) 15, 17, 19, 34, 25, 26, 27, 30-36 even, 40-42, 44-47.